Tutorial Note # 1
Important RF & MW Parameters for Broadband Communication
At RF and microwave frequencies scattering parameters are very common. This is the general definition of the S-parameters for an N-port with different characteristic impedances. The requirement that all other input voltages have to be zero also means that there are no re-reflections on any port.

If our device is a two port, and if we have the same characteristic impedance on input and output, the definition of the S-parameters is very easy. Please note that the requirement that all other input voltages have to be 0 also means that there are no signals reflected on any port!
Frequency response (amplitude and phase response) is the Fourier transform of the impulse response; the impulse response is the derivative of the step response. Shown here is the effect that different frequency responses have on a pulse: The black and the red trace have similar bandwidth, but the red trace has a steeper roll off, therefore the step response shows more ringing. The blue trace has the same shape than the black, but less bandwidth, therefore the rise time is slower. Keep in mind that not only the bandwidth is important for good pulse response, but also a linear phase response.

It might not be obvious at a first glance why a linear phase response is important. Let's have a look why this is especially important for broadband data signals: The square wave signal in our example is represented by the following equation:

\[ U(t) = 1 \cdot \sin(\omega_1 \cdot t) + \frac{1}{3} \cdot \sin(3 \cdot \omega_1 \cdot t) + \frac{1}{5} \cdot \sin(5 \cdot \omega_1 \cdot t) + \frac{1}{7} \cdot \sin(7 \cdot \omega_1 \cdot t) + \frac{1}{9} \cdot \sin(9 \cdot \omega_1 \cdot t) + \frac{1}{11} \cdot \sin(11 \cdot \omega_1 \cdot t) \]

The corresponding spectrum is also shown.

If we send this signal over an ideal transmission line, we will observe a delay of the signal at the output of the line. The delay will be: Length of line / Speed of propagation. Another way of specifying the delay is the phase:

\[ \varphi = \frac{\text{Length}_{\text{line}}}{\text{Wavelength}} \times 360^\circ \]

with: \( \text{Wavelength} = \frac{\text{Speed}_{\text{propagation}}}{\text{Frequency}} = \frac{c}{f} \)

we get:

\[ \varphi(l, f) = 360^\circ \cdot \frac{\text{Length}_{\text{line}} \cdot f}{c} \]
Another approach to understand the phase is if we look at the sine wave signals that make up our square wave: Since the delay of an ideal transmission line is the same for each sine wave, the phase has to be different for each frequency. This will lead to the same formula:

\[ \phi(t, f) = 360^\circ \cdot \frac{f}{c} \]

To characterize a signal in the frequency domain completely we have therefore to specify the amplitude and phase (or real and imaginary part) of each spectral component or frequency.

In our example, the fundamental frequency is rotated by 40°, the 3rd harmonic has thus a phase rotation of 120°, the 5th harmonic of 200°.

The phase response of our ideal transmission line is linear, therefore we do not observe any distortion of the transmitted signal.

The time domain signal would be represented by the following equation:

\[
U(t) = 1 \cdot \sin(\omega \cdot t - \phi) + \frac{1}{3} \cdot \sin(3 \cdot \omega \cdot t - 3 \cdot \phi) + \\
+ \frac{1}{5} \cdot \sin(5 \cdot \omega \cdot t - 5 \cdot \phi) + \frac{1}{7} \cdot \sin(7 \cdot \omega \cdot t - 7 \cdot \phi) + \\
+ \frac{1}{9} \cdot \sin(9 \cdot \omega \cdot t - 9 \cdot \phi) + \frac{1}{11} \cdot \sin(11 \cdot \omega \cdot t - 11 \cdot \phi)
\]
Let us have a look what will happen if we transmit a signal through a system with non-linear phase response: The two signals on the right have the same spectral content, however the phase of the spectral components relative to each other are different. In the upper right picture the phase of the spectral components is linear. The distortion of the signal in the lower right is only due to the non linear phase relation of the spectral components, the signal in this example is defined by the equation:

\[ U(t) = \sin(\omega \cdot t - \varphi) + \frac{1}{3} \sin(3 \cdot \omega \cdot t - 3 \cdot \varphi + 36^\circ) + \frac{1}{5} \sin(5 \cdot \omega \cdot t - 5 \cdot \varphi - 36^\circ) + \]
\[ + \frac{1}{7} \sin(7 \cdot \omega \cdot t - 7 \cdot \varphi) + \frac{1}{9} \sin(9 \cdot \omega \cdot t - 9 \cdot \varphi - 36^\circ) + \frac{1}{11} \sin(11 \cdot \omega \cdot t - 11 \cdot \varphi - 72^\circ) \]

It is important to mention that a non linear phase response not only causes distortion but also contributes to jitter.

Now that we know why linear phase response is important we can introduce two new terms: Deviation from linear phase and group delay:
At high frequencies, even short devices cause a phase rotation by many degrees, so the slope of the phase response will be very steep. Therefore it will be difficult to judge whether the phase response is linear or not.

One solution to this problem is to subtract a linear phase response from the measured phase response. The remaining non-linear part can then be displayed and analyzed with high resolution.

Another solution is to determine the derivative of the phase versus frequency. This leads to the group delay: The group delay is defined as: \( T_G = -\frac{d\phi}{d\omega} = -\frac{d\phi}{2\pi f} \)

As a network analyzer measures at discrete frequencies the group delay will be approximated by: \( -\frac{\Delta\phi}{2\pi \Delta f} \), the term \( \Delta f \) is called the aperture. It is very important that you use the same aperture if you compare measurements, as a bigger aperture will smooth the group delay and lead to “better” results!
VSWR... Voltage Standing Wave Ratio
The original method of measuring impedance or match was to use a slotted line, where a detector measured the voltage maximum and the voltage minimum (as a function of the position) of the standing wave caused by any mismatch on the output of the line.
Although today a vector network analyzer can measure the reflection coefficient directly, the VSWR is still a very common specification.

If a line is terminated with a device having an $s_{11}$ that is not zero, a part of the incident wave will be reflected. If the source has an impedance that is different than the characteristic impedance of the line, a part of the reflected signal will be re-reflected. Incident waves and re-reflected waves will add as vectors and this will cause ripple in the frequency domain.

Also in the time domain, reflections cause disturbances: If we send a pulse down the line the re-reflection will add to the original pulse. We will observe overshoot and ringing on the pulse. Shown in this slide is the pulse response for three different terminations:

i) As a short can not absorb energy, the entire signal is reflected.
ii) As the voltage across a short is zero, the reflection cancels the incident signal, i.e. the reflection has a relative phase of 180° to the incident signal.
iii) With a $Z_0$ termination, the incident signal is absorbed in the load and no reflections occur.

If we terminate with an open, the reflection will be in phase with the incident signal, therefore the voltage across the open will double.
There are many parameters characterizing match or mismatch: Impedance, reflection coefficient, $S_{11}$, VSWR and return loss. Shown in this slide is the relation between these parameters. Keep in mind that impedance and reflection coefficient ($S_{11}$) are vectors, whereas return loss and VSWR are scalar parameters.

It is sometimes confusing as the parameters have of course different values for match and mismatch:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Match</th>
<th>Short</th>
<th>Open</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z$</td>
<td>$Z_0$</td>
<td>0</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>0</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>VSWR</td>
<td>1</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$a_r$</td>
<td>$\infty$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Since the s-parameters are only valid for the linear region of the device - or for non-linear devices only for a particular input power - the non-linear behavior has to be specified as well.

A common specification is the 1dB compression point. This compression point is the output power at which the output power is one dB less than the output power of an ideal (linear) amplifier with the same gain.

Normally amplifiers are used in their linear region to avoid distortion of the signal, but for some applications the non-linear behavior is helpful: The limiting characteristic of an amplifier can be used to clip overshoot and ringing of data signals. In this case even higher compression (2dB or 3dB) is used.

Another specification of active components and systems is the noise figure: The noise figure is a measure of how much noise is added by e.g. an amplifier or in other words how much the signal to noise ratio of the input signal is degraded at the output.

If the input is terminated with $50 \, \Omega$, the noise at the output will be:

$$-174 \text{dBm/Hz} + 10 \log_{10} (\text{BW}) + G/\text{dB} + \text{NF/\text{dB}}$$
As mentioned the noise figure is the ratio of the input- and output- signal to noise ratio:

\[ F = \frac{(S/N)_{\text{Input}}}{(S/N)_{\text{Output}}} \]

and: \( NF/dB = 10 \log (F) \)

As the noise power density of a 50 Ohm resistor is given by:

\[ P_{\text{Noise}} = kT \]

where:

- \( k \) ….Boltzman Konstant \( (1.38 \times 10^{-23} \text{ J/K}) \)
- \( T \) ….Absolute Temperature \( / \text{°K} \)
- \( B \)…. Bandwidth / Hz

The output noise power can be calculated to be:

\[ P_{\text{Noise}}/W = kTBGF \]

\( G \)…. Gain (in linear terms!)

The noise figure can be expressed in linear terms or in dB.

Sometimes the noise temperature is used instead of the noise figure: The noise temperature is the theoretical temperature of the input resistor that would give the same noise power at the output of a noise free network than the “real world” network.

The relation between \( F \) and \( T_N \) is:

\[ T_N = (F-1)T_0 ; \text{ where } T_0 = 290^\circ \text{ K}. \]

Another application for broadband amplifiers is to use them as preamplifiers for receivers or spectrum analysers. How does the sensitivity improve when we use a preamplifier?

According to the cascaded noise equation the noise figure of two cascaded systems is:

\[ F_{\text{System}} = F_1 + \frac{F_2 - 1}{G_1} \]

With \( F_1 = 6.3 \ (10^{10}) \), \( F_2 = 3162 \ (10^{15/10}) \) and \( \text{Gain} = 398 \ (10^{26/10}) \) we get:

\( F_{\text{System}} = 14.25 \) and \( NF_{\text{System}} = 11.5 \text{ dB} \)

In our example the use of the pre-amplifier would improve the sensitivity of the spectrum analyser from \(-139\text{dBm/Hz}\) to \(-162\text{dBm/Hz}\).
For broadband HEMT amplifiers the noise figure varies with frequency. The noise figure increases especially below 100 kHz.

Note that the noise figure in the above diagram the frequency scale is logarithmic, when the frequency scale is linear the peak at low frequencies is not that obvious.

As the noise output power of our broadband amplifiers is far below one μW and as the signal output power is normally between 10 mW and 100 mW, noise is not an issue when amplifying broadband communication signals.

Noise in Time Domain

\[ \text{Noise}_{\text{time domain}} = \int \text{Noise figure}(f) \, df \]

Noise seen on an oscilloscope is proportional to the area below the noise figure versus frequency graph.

The noise figure peak at low frequencies is neglectable if you have 10 GBit/s and 40 GBit/s signals!